

6.3.1 EXERCISES

For a link to all of the additional resources available for this section, click [OSttS Chapter 6 materials](#).

In Exercises 1 - 33, solve the equation analytically.

For help with these exercises, click on the resource below:

- [Solving equations involving exponential functions](#)

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| 1. $2^{4x} = 8$ | 2. $3^{(x-1)} = 27$ | 3. $5^{2x-1} = 125$ |
| 4. $4^{2x} = \frac{1}{2}$ | 5. $8^x = \frac{1}{128}$ | 6. $2^{(x^3-x)} = 1$ |
| 7. $3^{7x} = 81^{4-2x}$ | 8. $9 \cdot 3^{7x} = \left(\frac{1}{9}\right)^{2x}$ | 9. $3^{2x} = 5$ |
| 10. $5^{-x} = 2$ | 11. $5^x = -2$ | 12. $3^{(x-1)} = 29$ |
| 13. $(1.005)^{12x} = 3$ | 14. $e^{-5730k} = \frac{1}{2}$ | 15. $2000e^{0.1t} = 4000$ |
| 16. $500(1 - e^{2x}) = 250$ | 17. $70 + 90e^{-0.1t} = 75$ | 18. $30 - 6e^{-0.1x} = 20$ |
| 19. $\frac{100e^x}{e^x + 2} = 50$ | 20. $\frac{5000}{1 + 2e^{-3t}} = 2500$ | 21. $\frac{150}{1 + 29e^{-0.8t}} = 75$ |
| 22. $25\left(\frac{4}{5}\right)^x = 10$ | 23. $e^{2x} = 2e^x$ | 24. $7e^{2x} = 28e^{-6x}$ |
| 25. $3^{(x-1)} = 2^x$ | 26. $3^{(x-1)} = \left(\frac{1}{2}\right)^{(x+5)}$ | 27. $7^{3+7x} = 3^{4-2x}$ |
| 28. $e^{2x} - 3e^x - 10 = 0$ | 29. $e^{2x} = e^x + 6$ | 30. $4^x + 2^x = 12$ |
| 31. $e^x - 3e^{-x} = 2$ | 32. $e^x + 15e^{-x} = 8$ | 33. $3^x + 25 \cdot 3^{-x} = 10$ |

In Exercises 34 - 39, solve the inequality analytically.

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| 34. $e^x > 53$ | 35. $1000(1.005)^{12t} \geq 3000$ |
| 36. $2^{(x^3-x)} < 1$ | 37. $25\left(\frac{4}{5}\right)^x \geq 10$ |
| 38. $\frac{150}{1 + 29e^{-0.8t}} \leq 130$ | 39. $70 + 90e^{-0.1t} \leq 75$ |

In Exercises 40 - 45, use your calculator to help you solve the equation or inequality.

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| 40. $2^x = x^2$ | 41. $e^x = \ln(x) + 5$ | 42. $e^{\sqrt{x}} = x + 1$ |
|-----------------|------------------------|----------------------------|

43. $e^{-x} - xe^{-x} \geq 0$ 44. $3^{(x-1)} < 2^x$ 45. $e^x < x^3 - x$
46. Since $f(x) = \ln(x)$ is a strictly increasing function, if $0 < a < b$ then $\ln(a) < \ln(b)$. Use this fact to solve the inequality $e^{(3x-1)} > 6$ without a sign diagram. Use this technique to solve the inequalities in Exercises 34 - 39. (NOTE: Isolate the exponential function first!)
47. Compute the inverse of $f(x) = \frac{e^x - e^{-x}}{2}$. State the domain and range of both f and f^{-1} .
48. In Example 6.3.4, we found that the inverse of $f(x) = \frac{5e^x}{e^x + 1}$ was $f^{-1}(x) = \ln\left(\frac{x}{5-x}\right)$ but we left a few loose ends for you to tie up.
- (a) Show that $(f^{-1} \circ f)(x) = x$ for all x in the domain of f and that $(f \circ f^{-1})(x) = x$ for all x in the domain of f^{-1} .
- (b) Find the range of f by finding the domain of f^{-1} .
- (c) Let $g(x) = \frac{5x}{x+1}$ and $h(x) = e^x$. Show that $f = g \circ h$ and that $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$. (We know this is true in general by Exercise 31 in Section 5.2, but it's nice to see a specific example of the property.)
49. With the help of your classmates, solve the inequality $e^x > x^n$ for a variety of natural numbers n . What might you conjecture about the “speed” at which $f(x) = e^x$ grows versus any polynomial?

Checkpoint Quiz 6.3

1. Solve for x : $2^{3x+1} = 7^{1-x}$
2. Solve for t : $10 - 3e^{-t} \leq 9$

For worked out solutions to this quiz, click the link below:

- [Quiz Solution](#)

6.3.2 ANSWERS

1. $x = \frac{3}{4}$
2. $x = 4$
3. $x = 2$
4. $x = -\frac{1}{4}$
5. $x = -\frac{7}{3}$
6. $x = -1, 0, 1$
7. $x = \frac{16}{15}$
8. $x = -\frac{2}{11}$
9. $x = \frac{\ln(5)}{2\ln(3)}$
10. $x = -\frac{\ln(2)}{\ln(5)}$
11. No solution.
12. $x = \frac{\ln(29)+\ln(3)}{\ln(3)}$
13. $x = \frac{\ln(3)}{12\ln(1.005)}$
14. $k = \frac{\ln(\frac{1}{2})}{-5730} = \frac{1}{5730} \ln(2)$
15. $t = \frac{\ln(2)}{0.1} = 10 \ln(2)$
16. $x = \frac{1}{2} \ln\left(\frac{1}{2}\right) = -\frac{1}{2} \ln(2)$
17. $t = \frac{\ln(\frac{1}{18})}{-0.1} = 10 \ln(18)$
18. $x = -10 \ln\left(\frac{5}{3}\right) = 10 \ln\left(\frac{3}{5}\right)$
19. $x = \ln(2)$
20. $t = \frac{1}{3} \ln(2)$
21. $t = \frac{\ln(\frac{1}{29})}{-0.8} = \frac{5}{4} \ln(29)$
22. $x = \frac{\ln(\frac{2}{5})}{\ln(\frac{4}{5})} = \frac{\ln(2)-\ln(5)}{\ln(4)-\ln(5)}$
23. $x = \ln(2)$
24. $x = -\frac{1}{8} \ln\left(\frac{1}{4}\right) = \frac{1}{4} \ln(2)$
25. $x = \frac{\ln(3)}{\ln(3)-\ln(2)}$
26. $x = \frac{\ln(3)+5\ln(\frac{1}{2})}{\ln(3)-\ln(\frac{1}{2})} = \frac{\ln(3)-5\ln(2)}{\ln(3)+\ln(2)}$
27. $x = \frac{4\ln(3)-3\ln(7)}{7\ln(7)+2\ln(3)}$
28. $x = \ln(5)$
29. $x = \ln(3)$
30. $x = \frac{\ln(3)}{\ln(2)}$
31. $x = \ln(3)$
32. $x = \ln(3), \ln(5)$
33. $x = \frac{\ln(5)}{\ln(3)}$
34. $(\ln(53), \infty)$
35. $\left[\frac{\ln(3)}{12\ln(1.005)}, \infty\right)$
36. $(-\infty, -1) \cup (0, 1)$
37. $\left(-\infty, \frac{\ln(\frac{2}{5})}{\ln(\frac{4}{5})}\right] = \left(-\infty, \frac{\ln(2)-\ln(5)}{\ln(4)-\ln(5)}\right]$
38. $\left(-\infty, \frac{\ln(\frac{2}{377})}{-0.8}\right] = \left(-\infty, \frac{5}{4} \ln\left(\frac{377}{2}\right)\right]$
39. $\left[\frac{\ln(\frac{1}{18})}{-0.1}, \infty\right) = [10 \ln(18), \infty)$
40. $x \approx -0.76666, x = 2, x = 4$
41. $x \approx 0.01866, x \approx 1.7115$
42. $x = 0$
43. $(-\infty, 1]$
44. $\approx (-\infty, 2.7095)$
45. $\approx (2.3217, 4.3717)$
46. $x > \frac{1}{3}(\ln(6) + 1)$
47. $f^{-1} = \ln\left(x + \sqrt{x^2 + 1}\right)$. Both f and f^{-1} have domain $(-\infty, \infty)$ and range $(-\infty, \infty)$.